A New Wavelet-Based Technique For Fast Full-Wave Physical Simulations of Millimeter-Wave Transistors

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Abstract — **A new wavelet-based simulation approach for the global modeling of high-frequency transistors is presented. The proposed approach solves the active device model that combines the transport physics and Maxwell's Equations on nonuniform self-adaptive grids. The nonuniform grids are obtained by applying Daubechies wavelet transforms followed by thresholding. This allows forming fine and coarse grids in locations where variable solutions change rapidly and slowly, respectively. The developed technique is validated by simulating a submicrometer transistor. Different numerical examples are presented along with illustrative comparison graphs, showing more than 75% reduction in CPU time, while maintaining the same degree of accuracy achieved using a uniform grid case. To the extent of the authors' knowledge, this is the first time in literature to implement and report a unified wavelet technique for fast full-wave physical simulations of millimeter-wave transistors.**

I. Introduction

ACCURATE modeling of high-frequency active devices should involve solving the equations that describe the transport physics in conjunction with Maxwell's Equations. This approach has been addressed by global circuit modeling that has been demonstrated in [1].

Despite being the correct way to model high-frequency devices, global modeling techniques suffer from their extensive CPU-time requirements [1]. Therefore, there is an imperative need to present a new approach to reduce the simulation time, while maintaining the same degree of accuracy presented by the global modeling techniques. A possible approach is to use multiresolution nonuniform grids. Such technique could be implemented using wavelets.

Accordingly, a fast global modeling simulation approach for high-frequency active devices should involve a unified technique to simulate both passive structures and active devices efficiently, using wavelets.

In literature, different wavelet-based simulation approaches have been developed for passive structures and active devices independently. For instance, various wavelet approaches have been successfully applied to finite-difference time-domain (FDTD) simulations of passive structures [2]-[3]. However, for the active devices that are characterized by a set of coupled and highly nonlinear partial differential equations (PDE's), applying the same approach would become quite time consuming [4]. On the other hand, interpolating wavelets have been successfully applied to the simple drift diffusion active device model [5]. Being primarily developed for long-gate devices, the drift diffusion model leads to inaccurate estimations of device internal distributions and microwave characteristics for submicrometer devices [6]. It is worth mentioning that in [5], the authors proposed a new technique to solve simple forms of Hyperbolic PDE's, using an interpolating wavelet scheme. These PDE's can represent Maxwell's Equations or the simple drift-diffusion model, but not the complete hydrodynamic model. Thus, a new approach to apply wavelets to the hydrodynamic model is needed along with extending it to Maxwell's Equations for a fast global modeling simulation approach of high-frequency active devices.

In this paper, a unified approach to apply wavelets to the full hydrodynamic model and Maxwell's equations is developed. The basic idea is to take snapshots of the solution during the simulation, and apply wavelet transform to the current solution to obtain the coefficients of the details. The coefficients of the details are then normalized, and a threshold is applied to obtain a nonuniform grid. Two independent grid-updating criteria are developed for the active and passive parts of the problem. Moreover, a threshold formula that is dependent on the variable solution at any given time has been developed and verified. In additions, problems related to boundary conditions and discritization are solved.

II. Methodology

The active device model is based on the moments of Boltzmanns Transport Equations obtained by integrating over the momentum space. The integration results in a strongly coupled highly nonlinear set of partial differential equations, called the conservation equations. These equations provide a time-dependent self-consistent solution for carrier density, carrier energy, and carrier momentum, respectively, and are given as

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \boldsymbol{v}) = 0 \quad (1)$$

$$\frac{\partial (n\varepsilon)}{\partial t} + qn\boldsymbol{v}\cdot \boldsymbol{E} + \nabla \cdot (n\boldsymbol{v}(\varepsilon + k_B T)) = -\frac{n(\varepsilon - \varepsilon_o)}{\tau_\varepsilon(\varepsilon)} \quad (2)$$

$$\frac{\partial (np_x)}{\partial t} + qnE_x + \nabla \cdot (np_x \boldsymbol{v}) + \frac{\partial (nk_B T)}{\partial x} = -\frac{n(p_x - p_o)}{\tau_m(\varepsilon)} \quad (3)$$

The wavelet-based algorithm for the active device is presented in [7]. However, the algorithm presented in [7] does not provide an efficient and adaptive threshold formula, which is proposed in this paper.

It should be noted that magnitude ranges of the variables used in the simulations vary dramatically. Accordingly, the threshold value should be dependent on the variable solution at any given iteration. The proposed threshold formula is given by Eq. (4).

$$T = T_0 \frac{\sqrt{\sum_{i=1}^{n_x} d_i^2}}{n_x} \quad (4)$$

In this equation, T_0 is the initial threshold value, d_i's are the coefficients of the details, and n_x is the number of grid points in x direction. In this manner, the value of the threshold T depends mainly on the variable solution at any given time, rather than being fixed.

The nonuniform grids are conceived by applying Daubechies wavelet transform to the variable solution at any given time to obtain the coefficients of the details, which are then normalized to its maximum. Only grid points where the values of the normalized coefficients of the details larger than the threshold value given by Eq. (4) are included for the next iteration. Fig. 1 demonstrates examples for the procedure employed to obtain the nonuniform grids for the passive and active parts of the problem. Considering Fig. 1, one can observe that the proposed algorithm accurately removes grid points in the locations where variable solutions change slowly.

Now, we turn our attention to Maxwell's Equations. The passive part of the FET represents a co-planar structure, in which a 3D FDTD is developed to solve for the electric and magnetic fields using Maxwell's Equations.

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \quad (5)$$

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{J} \quad (6)$$

The current density estimated from the active device conversation equations is used to update the fields in Maxwell's Equations. It is worth to mention that the same approach developed to obtain the nonuniform grid for the

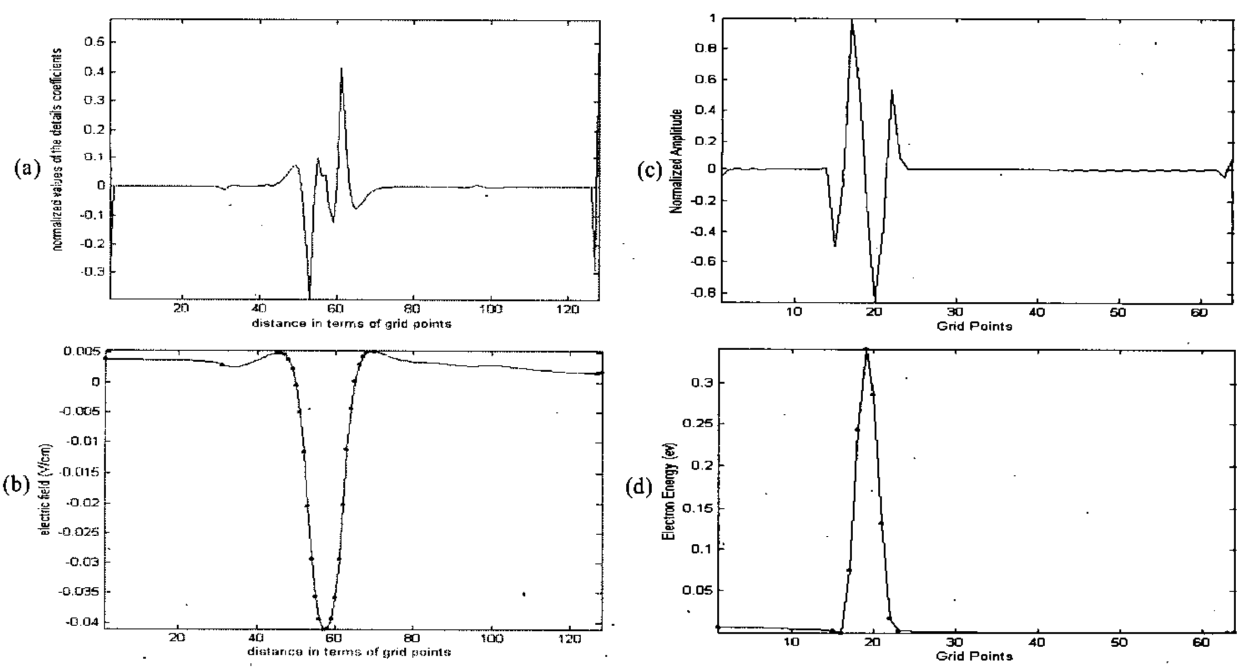

Fig. 1. (a) Normalized details coefficients for the electric field of the passive part. (b) Grid points marked on the actual curve of the electric field. (c) Normalized details coefficients for the electron energy. (d) Grid points marked on the actual curve of the electron energy.

variables in the conversation equations is applied to Maxwell's Equations as well. However, a different updating mechanism should be developed to keep track of the wave propagation within the passive part. The following is the algorithm developed for grid updating of FDTD simulations.

Step 1: Construct a 3D matrix M that has only 0's and 1's, based whether or not a non-zero solution of the field exists at this location. For instance, "1" is assigned if a non-zero field solution exists, and "0" elsewhere.

Step 2: Estimate the value of ρ (FDTD grid-updating factor) as

$$\rho = \frac{\sum_{i,j,k} (M_{new} \oplus M_{old})_{i,j,k}}{N_{xd} \cdot N_{yd} \cdot N_{zd}} \tag{7}$$

where M_{new} and M_{old} are the matrices constructed using step one for the current and old solutions of the fields, respectively. N_{xd}, N_{yd}, and N_{zd} are the number of grid points in x, y, and z directions, respectively.

Step 3: Check ρ's value against a predefined value, for example 5%.

Step 4: If satisfied, move the grid to $z = z + dz$, where dz is proportional to ρ.

Step 5: $t = t + dt$

III. Results And Discussions

A. Hydrodynamic Model Simulation Results

The approach presented in this paper is generic, which can be employed to simulate any unipolar transistor. To demonstrate the potential of this approach, it is applied to an idealized FET structure with the following dimensions: 0.3μm gate length, 0.5μm long source and drain electrodes, 0.5μm source-gate gap, 1μm gate-drain separation, 0.2μm deep channel layer and a 0.6μm deep buffer layer. The doping of the active layer is $2 \cdot 10^{17}\,\text{cm}^{-3}$ and the one of the buffer layer is $10^{14}\,\text{cm}^{-3}$. The transistor is discretized using a mesh size of 64Δx by 64Δy, with $\Delta t = 0.001\text{ps}$. Forward Euler is adopted as an explicit finite-difference method. In addition, upwinding is employed to have a stable finite-difference (FD) scheme. The space step sizes are adjusted to satisfy Debye length, while the time step value Δt is chosen to satisfy the Courant-Friedrichs-Levy (CFL) condition. The simulated device is biased to V_{ds} =3.0 and V_{gs} =-0.5V. The DC distributions are obtained by solving the active device model only, with the proposed algorithm employed.

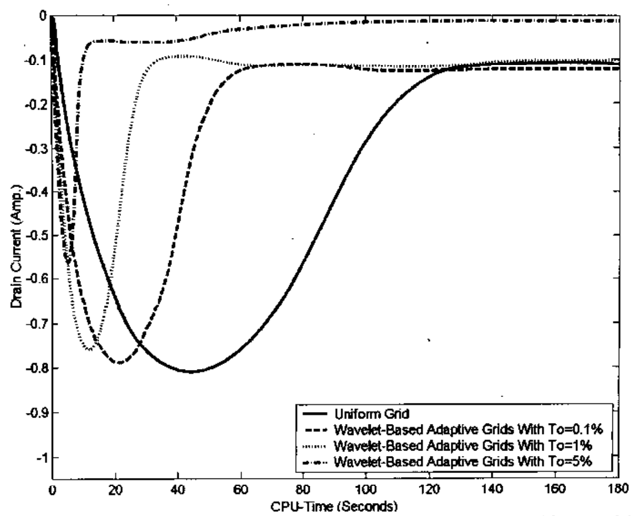

Fig. 2. DC drain current convergence curves for the uniform grid case and the proposed algorithm with different threshold values.

Considering Figures (2)-(3), it can be observed that using the proposed approach has reduced the CPU-time dramatically. For instance, there is a reduction of about 75% in CPU-time over the uniform grid case for an initial threshold value of 1%, while the DC drain current error is within 1%. Furthermore, using large initial threshold values affects the accuracy of the solution despite the CPU-time reduction. The reason is employing a large initial threshold value results in removing significant grid points, which degrades the final results.

Moreover, it is noticed that there is no significant difference in terms of accuracy between the two cases of initial threshold values equal to 0.1% and 1%. The mean relative error for the two cases is in the order of 3 ~ to 4%.

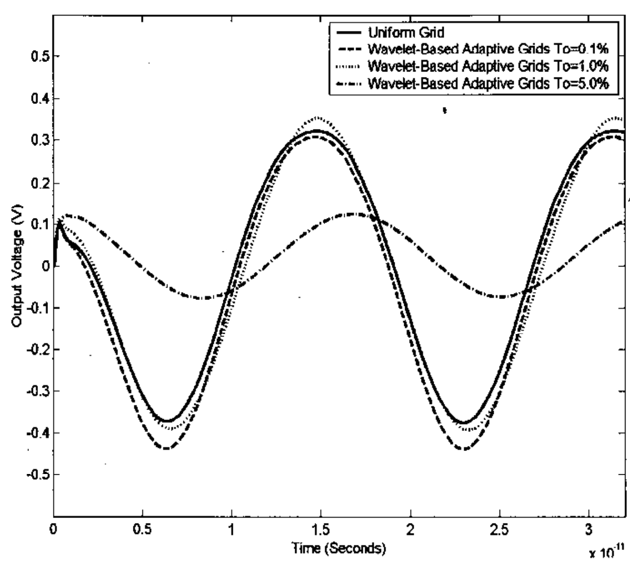

Fig. 3. AC output voltage for the uniform grid and the proposed algorithm for different values of the initial threshold.

This suggests that using initial threshold value equals to 1% be the right choice in terms of both accuracy and CPU-time.

B. *FDTD Simulation Results*

A 3-D Yee-based FDTD code is developed for the passive part of the problem, with the proposed algorithm employed. In addition, a Gaussian excitation pulse is applied to evaluate the algorithm over a wide range of frequencies. Table I depicts the results, where it is observed that as the threshold value increases, CPU-time and error introduced decreases as well. It is worth mentioning here that using an initial threshold value equals to 10% seems to reduce error along with the CPU-time. However, considering Fig. 4, one can conclude that using an initial threshold value equals to 10% introduces dispersion, which is a serious type of error. Therefore, an initial threshold value of 5% is recommended in terms of both CPU-time and error.

TABLE I

T_0	*CPU-time*	*Error on Potential*	
		2-norm	*Infinity-norm*
(Uniform Grid)	744.90 s		
0.1%	300.17 s	0.0873%	8.80%
1.0%	205.92 s	0.0871%	8.75%
5.0%	155.10 s	0.0778%	7.69%
10.0%	111.05 s	0.0473%	3.66%

It is important to emphasize that the passive and active parts of the problem have different optimal threshold values. This is expected since the variables in the conservations equations are highly nonlinear compared to the fields obtained when solving Maxwell's Equations.

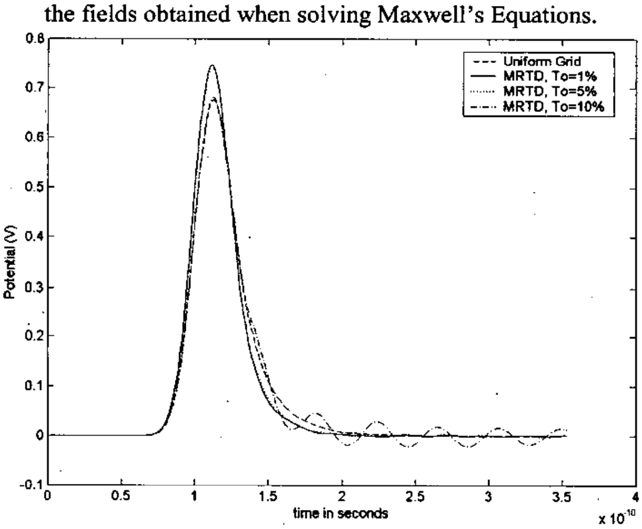

Fig. 4. Potential of the gate at a specific cross-section versus time for the uniform grid case and the proposed algorithm with different initial threshold values.

V. CONCLUSIONS

A new wavelet-based full-wave physical simulation approach has been developed and successfully applied to a millimeter-wave transistor. The proposed technique solves the PDE's that describe the transport physics, and Maxwell's Equations on nonuniform self-adaptive grids, obtained using a new wavelet-based technique. Moreover, efficient grid updating criteria for the active and passive parts of the problem have been developed and verified. A reduction of 75% in CPU-time is achieved compared to a uniform grid case with an error of 2% on the DC drain current, and a mean relative error of order 3 to 4% on the AC output voltage. Moreover, an 80% CPU-time reduction is obtained for FDTD simulations with approximately 0.1% average error on the potential. It has been observed that tradeoffs exist between the threshold value, CPU-time, and accuracy, suggesting an optimal value for the threshold.

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REFERENCES

[1] R. O. Grondin, S. M. El-Ghazaly, and S. Goodnick,"A review of global modeling of charge transport in semiconductors and full-wave electromagnetics," *IEEE Trans. Microwave Theory Tech.*, vol. 47, pp.817-829, June 1999.

[2] E. M. Tentzeris, A. Cangellaris, L. P. B. Katehi, and J. Harvey, "Multiresolution time-domain (MRTD) adaptive schemes using arbitrary resolutions of wavelets," *IEEE Trans. Microwave Theory Tech.*, vol. 50, pp.501-516, Feb. 2002.

[3] M. Fujii and W. J. R. Hoefer, "A three-dimensional Haar-Wavelet-based multiresolution analysis similar to the FDTD method- derivation and application, " *IEEE Trans. Microwave Theory Tech.*, vol. 46, pp.2463-2475, Dec. 1998.

[4] J. Keiser, " Wavelet based approach to numerical solution of non-linear partial differential equation," Ph.D. Thesis, University of Colorado, Boulder, 1995.

[5] S. Goasguen and S. M. El-Ghazaly, " Interpolating wavelet scheme toward global modeling of microwave circuits," in *IEEE MTT-S Int. Symp. Dig.*, 2000, pp.375-378.

[6] Y.K. Feng and A. Hints, "Simulation of submicrometer GaAs MESFET's using a full hydrodynamic model, " *IEEE Trans. Electron Devices*, vol. 35, pp.1419-1431, Sep. 1988.

[7] Y. A. Hussein and S. M. El-Ghazaly, " Global Modeling of Active Microwave Devices Incorporating a Novel Full Hydrodynamic Physical Simulator Using Wavelet-Based Adaptive Grids," in *IEEE MTT-S Int. Symp. Dig.*, 2002, pp.743-746.